High-Resolution Simulation of Granular Material with SPH

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Abstract

• **We present an efficient framework** for simulating granular material with high visual detail

• Our model solves the computationally and numerically critical forces on a *coarsely sampled particle simulation*

• Visual realism is achieved by coupling a set of highly resolved particles with the base simulation at **low computational costs**

• We incorporate a **new frictional boundary force**
Introduction: Character of Granular Materials

• They settle in stable piles and act like a solid
  ▪ if the average energy is low

• When freely flowing, they have similar characteristics as ordinary Newtonian fluids
  ▪ but granular material dissipates energy quickly

• The complex dynamics arise from the interplay of contact forces between the elements
Introduction: Present Trend of Granular Material Simulation

• Various simulation methods have been developed
  ▪ in the engineering field and in computer animation

• The main focus in animation is set on efficient techniques that achieve visually plausible results
  ▪ Simplifying assumptions
  ▪ Coarse discretization
  ▪ Avoids simulating each physical grain
  ▪ Rich visual detail
Introduction: The Main Contribution of This Paper

- Lagrangian simulation framework which captures granular dynamics realistically and uncovers **high visual detail at low computational costs**

- Appropriate mechanical behavior is modeled by computing **frictional and pressure forces on a coarse scale**

- High visual detail is obtained in a **post-process**
  - fine-scaled particles are coupled to the base simulation
Introduction: The Main Contribution (cont’)

• Upsampling method increases the visual quality dramatically without uncovering the base simulation
  ▪ even for large upscaling factors and scenarios with dynamic objects

The base simulation (left, 38K) and the secondary simulation (middle and right, 1.4M and 19.4M, left to right)
Related Work:
Two Methods of Granular Material Simulation

- There are two methods
  - Discrete
    - Particle-based method
    - The topology of particle interaction can evolve freely
    - Splashing and avalanches can be conveniently generated
    - Two-way coupling can be simulated easily

Particle-Based Simulation of Granular Materials
[Nathan Bell et al. / SCA 2005]
Related Work:
Two Methods of Granular Material Simulation (cont’)

- Continuum
  - Grid-based method
  - **Internal forces** are computed on a **coarse grid**.
  - The resulting **velocity field** is then used to **advect particles**.
Related Work: Previous Continuum Methods

- **Animating Sand as a Fluid**
  [ZHU Y. et al. / SIGGRAPH 2005]

- **Mixing fluids and granular materials**
  [LENAERTS T. et al. / CGF 2009]

- **Free-flowing granular materials with two-way solid coupling**
  [NARAIN R. et al. / SIGGRAPH Asia 2010]

- **SPH Granular Flow with Friction and Cohesion**
  [ALDUÁN I. et al. / SCA 2011]
Contribution

- Eliminate oscillations in the pressure field
  - Using physically-based rigid-fluid coupling
    - Versatile Rigid-Fluid Coupling for Incompressible SPH [AKINCI N. et al. / TOG 2012]

- Simulation of fined-grained material is implemented
  - Using High-Resolution particles in post-process

- Eliminate clumping artifacts
  - Using new approach
Algorithm Overview

[Images of geometric models and human figures]
Algorithm

- There are two steps
  - Granular Simulation Framework
    - Coarse scale simulation
    - Solid body interaction
  - Fine Scale Simulation
    - Sampling
    - Advection
Algorithm: Granular Simulation Framework

- Coarse Scale Simulation
  - Base simulation
    - The framework builds on the SPH-based continuum method
      - SPH Granular Flow with Friction and Cohesion
      - [ALDUÁN I. et al. / SCA 2011]

- Solid Body Interaction
  - Propose new friction model
Algorithm:
Coarse Scale Simulation (Pressure and Density)

- Density
  - Using basic SPH density term
    \[ \rho_i = m \sum_j W(x_{ij}, h), \quad (x_{ij} = x_i - x_j) \]

- Pressure
  - PCISPH term
    \[ F_i^P = -m_i \sum_j \left( \frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) \nabla W_{ij}, \quad \rho \leq \rho_0 \text{ and } p \geq 0, \text{ where either } \rho = \rho_0 \text{ or } p = 0 \]

Iterate \( p_i + = \tilde{p}_i. \)

\[ \tilde{p}_i = \frac{\Delta \rho_i(t)}{\beta(-\sum_j \nabla W_{ij} \cdot \sum_j \nabla W_{ij} - \sum_j (\nabla W_{ij} \cdot \nabla W_{ij}))} \]

where \( \beta \) is

\[ \beta = \Delta t^2 m^2 \rho_0^2. \]
Algorithm: Coarse Scale Simulation (Friction)

- Friction
  - Using friction & cohesion term
    - SPH Granular Flow with Friction and Cohesion
    - [ALDUÁN I. et al. / SCA 2011]

\[
F_i^f = -m_i \sum_j m_j \left( \frac{s_i}{\rho_i^2} + \frac{s_j}{\rho_j^2} \right) \nabla W_{ij}
\]

Iterate \( s_i += \Delta s_i \)

\[
\Delta s_i = D^{-1} \dot{\varepsilon}_{i,0}, \quad \text{with} \quad D = \frac{2m_i^2 \Delta t}{\rho_i^2} \left( \sum_j \frac{1}{\rho_j} \nabla W_{ij} \nabla W_{ij}^T \right), \quad -\dot{\varepsilon}_{i,0} = \sum_j \frac{m_j}{\rho_j} \nabla W_{ij} \Delta u_j^T
\]
Algorithm: 
Solid Body Interaction

• Propose new method
  ▪ Previous method makes perceivable artifacts
    • Oscillation in the pressure field
    • In order to this problem
      ▪ Use small time step
      ▪ The number of iteration has to be set high (i.e. larger than five)

▪ In order to oscillation problem, propose new method
  • Using the boundary handling method of
    ▪ Versatile Rigid-Fluid Coupling for Incompressible SPH
    ▪ [AKINCI N. et al. / TOG 2012]
Algorithm: 
Solid Body Interaction

- New term for friction
  - Density
    \[ \rho_i = \sum_j m_j W(x_i - x_j, h) + \sum_b \frac{\rho_0}{\delta_b} W(x_i - x_b, h), \quad \delta_b = \frac{1}{\sum_j W_{bj}} \]
  - Pressure
    - Act boundary particles to granular particles
      \[ F^p_{i \leftarrow b} = -m_i \Psi_b(\rho_0) \frac{p_i}{\rho_i^2} \nabla W_{ib}, \quad \Psi_b(\rho_0) \equiv \frac{\rho_0}{\delta_b} \]
  - Friction
    - Act boundary particles to granular particles
      \[ F^f_{i \leftarrow b} = -m_i \Psi_b(\rho_0) \frac{s_i}{\rho_i^2} \nabla W_{ib} \]
Algorithm:
Solid Body Interaction (cont’)

- Friction
  - Finally, use this term

\[
F_{i \leftarrow b}^f = \max(\|F_{i \leftarrow b}^v\|, \|F_{i \leftarrow b}^s\|)
\]

\[
F_{i \leftarrow b}^s = -m_i \Psi_b(\rho_0) \frac{S_i}{\rho_i^2} \nabla W_{ib}, \quad F_{i \leftarrow b}^v = -m_i \Psi_b(\rho_0) \Pi_{ib} \nabla W_{ib}
\]

where \( \Pi_{ib} = -\frac{\sigma_{ib} h c_s}{2 \rho_i} \left( \frac{\min(v_{ib} \cdot x_{ib}, 0)}{|x_{ib}|^2 + \varepsilon h^2} \right) \)
Algorithm: Fine Scale Simulation

- Sampling
  - In order to avoid aliasing and distortions
    - Random Sampling Method

- Advection
  - Advection method should not
    - Compute internal force
    - Perform collision tests between HR particles

- HR particles follow some behaviors
  - follow the mechanical flow of the base simulation
  - should be allowed to disperse freely
  - should smoothly align without forming clumps
Algorithm: Sampling

- Sampling
  - LR(low-resolution) particle has bounding box
  - Each bounding box has seven sampling points
    - One at the particle center
    - One at each intersection point of the bounding box
  - HR(high-resolution) particles are randomly sampled
    - Around each sampling points
      - In a cubical volume
Algorithm: Advection

• Advection
  ▪ New approach
    • The method of previous work has limitation
      ▪ HR particles have the interpolated velocity of LR particle
        • When HR particles have one LR particle or
        • Are only influenced by external force
      ▪ This method avoids the clumping problem only if $h_{HR} \approx r_{LR}$

• In order to improve this limitation
  ▪ Blend the contributions of the base simulation and external forces
Algorithm: Advection (cont’)

- Advection
  - Compute distance-based weights
    - $j$ is LR particle or boundary particle
  
  $$w(d_{ij}) = \max \left[ 0, \left( 1 - \frac{d_{ij}^2}{h_{HR}^2} \right)^3 \right]$$
  where $d_{ij} = |x_i - x_j|$

- Compute average velocity

  $$v_i^*(t + \Delta t) = \frac{1}{\sum_j w(d_{ij})} \sum_j w(d_{ij}) v_j$$

- Final velocity

  $$v_i(t + \Delta t) = (1 - \alpha_i) v_i^*(t + \Delta t) + \alpha_i \left( v_i(t) + \Delta t \frac{F^g}{m} \right), \quad \alpha_i = \begin{cases} 
  1 - \arg \max_j w(d_{ij}) & \frac{\arg \max_j w(d_{ij})}{\sum_j w(d_{ij})} \geq 0.6, \\
  1 - \arg \max_j w(d_{ij}) & \arg \max_j w(d_{ij}) \leq w(r_{LR}) \\
  0 & \text{otherwise.} \end{cases}$$
Result

Varying Visual Detail

38K base particles

1.4M HR particles

19.4M HR particles
Conclusion

• Solve the clumping problem

• Propose friction-based coupling model
  ▪ more realistic

• More robust

• Can use very large upscaling factors
Limitations and Future Work

• Limitations
  ▪ The proposed coupling does not model static friction correctly
  ▪ The proposed HR particle advection has compression problem

• Future Work
  ▪ The granular material could be coupled with an SPH fluid
    • In order to erosion effects
    • Transitions from dry to moist sand and mud