Noname manuscript No.
(will be inserted by the editor)

Interchangeable SPH and Level set Method In Multiphase Fluid

Ho-Young Lee · Jeong-Mo Hong · Chang-Hun Kim

Received: date / Accepted: date

Abstract In grid-based fluid simulation, subgrid-scale fluid is difficult to represent realistically. We have come to describe such small scale details effectively, even on a coarse grid by using escaped particles. These particles, when simulated through smooth particle hydrodynamics, allow the illustration of dynamic and realistic fluids. Particles in SPH have a force for them to be merged within the range of each particle's radius. However, the SPH approach does not address the objective of only the part indescribable on a grid being portrayed through the particle method, decreasing the accuracy of the simulation. Integrated particles which are able to be described by level set method are also likely to end up simulated by particles. To address this problem, this paper introduces method through which the indefinable part of a grid is denoted in particles, while level set method is used to describe the particles merged on the grid.

Keywords Fluid simulation · Physically based modeling · Bubbles · SPH · level set · Grid-based simulation · Multiphase fluids

1 Introduction

Fluids research in the computer graphics community has largely been focused upon the precision and realistic visual quality of simulations. Advection of simulation is also indispensable for greater accuracy, while BFECC [14] and CIP [22, 21] ensure second order accuracy. We adopted the BFECC method to improve simulation accuracy and implemented simulation using the particle level set method [6, 7] to detect smooth surfaces. Even with both methods applied, Eulerian grid-based simulation, which is based

Ho-Young Lee, Chang-Hun Kim
Dept of Computer Science, Korea Univ., Anam-dong Seongbuk-Gu, Seoul, KOREA
Tel.: +82-2-3290-3574
Fax: +82-2-929-1917
E-mail: {flymist, chkim}@korea.ac.kr

Jeong-Mo Hong
Dept of Computer Science, Dongguk Univ., Pil-dong Jung-Gu, Seoul, KOREA
Tel.: +82-2-2260-3826
E-mail: jmhong@dongguk.edu

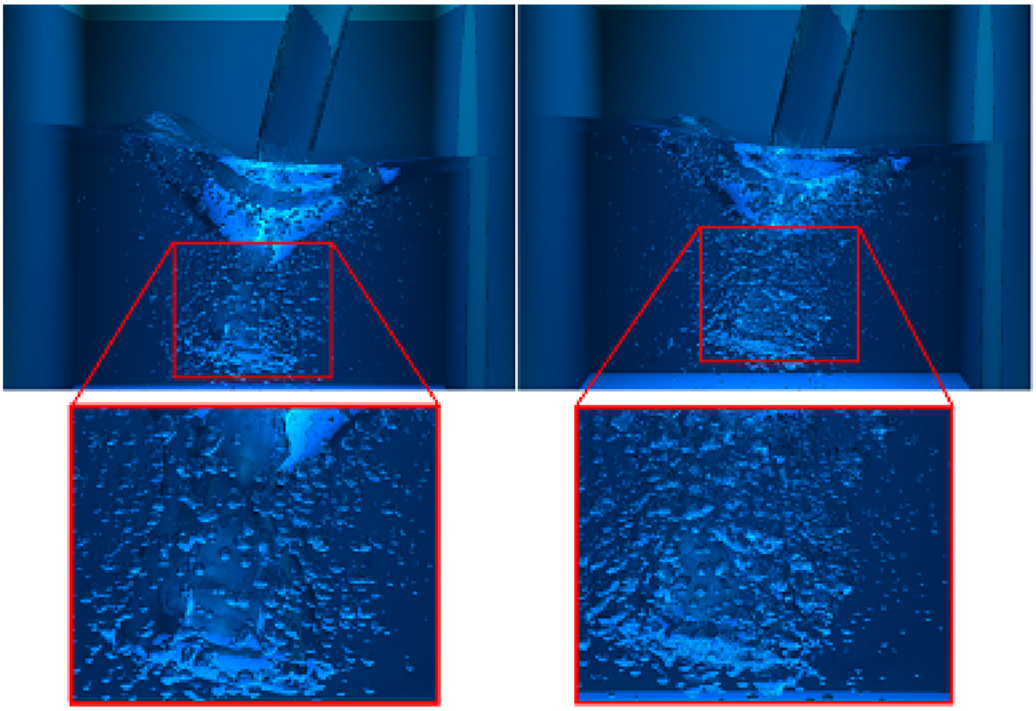

Fig. 1 Exmaple of pouring water. All the bubbles are simulated as SPH particles using [12] method on the left while right includes our method of describing large air bubbles in level set and smaller ones as SPH particles.

on Navier-Stokes equations, continues to result in dissipation. Currently, a great deal of research on this problem is underway. One outstanding example, and a primary motive for this paper, is [12]. The combination of particle and grid-based methods boosts realistic visual attributes and, most notably, plays an important role in depicting splashing, foam, and bubbles. Generally, the particles remaining outside the level set surface after the correction of the particle level set method are utilized. Meanwhile, this research is being conducted to improve authenticity through the addition of SPH-based physical properties. We also performed SPH-adopting simulation for the remaining escaped particles. Then, particles and grid were attempted two-way coupling using physical dynamics. Also, vorticity confinement method and cohesive force were added to SPH particle to harmonize grid-based simulation and particle simulation. Cohesive force refers to a force through which particles draw one another together. This force attracts neighboring particles and eventually engenders particle clustering. Prior research has adopted a hybrid approach for particle and grid-based simulation in order to exhibit the indefinable part on a grid. Therefore, this paper will focus on method to improve the accuracy of simulation. To this end, we utilize SPH particles to display the indefinable on the grid scale while small-scale features and particles which are clustered and become large enough to be depicted via the grid method are converted to level set on a grid.

2 Previous Work

Foster and Metaxas [8] made three dimensional fluid simulation based on Navier-Stokes equations and Stam [20] suggested a semi-Lagrangian integration scheme and introduced unconditionally stable fluid. Authors of [6, 7] adopted particle and level set and pioneered tracking complicated water surfaces. In [10], the volume of fluid method shows multiphase fluid simulation of surface tension between water and air bubble. Subsequently, Hong and Kim [11] suggests a numerical method that emphasizes a discontinuous interface among different fluids. Desbrun and Cani [5] focuses the SPH method to handle viscous fluids and Müller et al. [17] proposes an interactive method, an underpinning of SPH in the simulation of water and represents a multiphase SPH method to describe fluids with different compositions [19]. Cleary et al. [4] treats the collision motion of foam and bubbles on a complicated surface realistically. Recently, great effort is being made to highlight the details of an underlying subgrid, facilitated by the hybridization of a Eulerian grid and Lagrangian particle. Greenwood and House [9] portrays small scale features of a subgrid on a grid-based simulation by adding the particle level set method [6], which uses the escaped marker particle from water or air. Kime et al. [13] also depicted splash by combining SPH method with escaped particles from particle level set. Losasso et al. two-way coupled particle level set and SPH [16] in order to simulate diffuse regions such as splashing. Finally, Hong et al. [12], the fundamental foundation of this paper, utilized SPH air bubbles on a coarse grid to create lively air bubbles with small-scale features.

3 Fluid Simulation

3.1 Grid-based Fluid Simulation

The Navier-Stokes equation provides the simulation of incompressible fluid, and preserves mass and momentum.

$$u_t = -(u \cdot \nabla)u + \nabla p/\rho = f \tag{1}$$

$$\nabla \cdot u = 0 \tag{2}$$

Where u={u,v,w} is the velocity, ρ is the density, and f is external force(such as gravity, buoyancy etc.). Equation (1) and (2) are solved by using Chorin's method [3]. We divide equation (1) into two equations by intermediate velocity u*.

$$\frac{u^* - u^n}{\Delta t} = -(u^n \cdot \nabla)u^n + f \tag{3}$$

$$\frac{u^{n+1} - u^*}{\Delta t} = -\frac{\nabla p}{\rho} \tag{4}$$

To obtain u*, first, we should define $-(u^n \cdot \nabla)u^n$, the advection term, from the semi-Lagrangian method [20], and the rest of the equation, excluding pressure, is added. Next, we make equation (4) Poisson's equation in equation (5) and yield the pressure profile, based on the divergence-free condition from equation (2).

$$\nabla^2 p = \frac{\rho}{\Delta t} \nabla \cdot u^* \tag{5}$$

Substitute pressure value in equation (6) to generate the velocity of the next step.

$$u^{n+1} = u^* - \frac{\Delta t}{\rho} \nabla p \tag{6}$$

We use the octree [15], focused on the interface. BFECC(Back and forth Error Compensation and Correction) method reduces easily volume loss; so we implement BFECC method to satisfy second order accuracy. Furthermore, we have found smooth surface from complicated water surface through the particle level set method [6,7].

3.2 SPH Fluid Simulation

SPH is a technique to interpolate the particle system. The equation to obtain $A(x_i)$ at a particle x_i is following:

$$A(x) = \sum_j m_j \frac{A_j}{\rho_j} W(x_{ij}, h) \tag{7}$$

Where ρ is the density of particle i; $W(x_{ij}, h)$ is a smoothing kernel with core radius h. m_j is the mass of particle j. The acceleration of particle i is calculated by dividing particle force f_i with density. Acceleration and velocity are used to yield velocity and position.

$$a_i = \frac{dv_i}{dt} = \frac{f_i}{\rho_i} \tag{8}$$

Adams et al. suggests the adaptive sampling algorithms [1]. In order to describe a variety of bubbles we use the radius adaptive. Pressure force is provided as in the following equation.

$$f_{ij}^{pressure} = -V_i V_j (P_i + P_j)(\nabla W(x_{ij}, r_i) + \nabla W(x_{ij}, r_j))/2 \tag{9}$$

Where the volume V_i is m_i/ρ_i, r is the radius, the mass m_i is proportional to r_i^3, $x_{ij} = x_j - x_i$, and the pressure $P_i = k\rho_i$ with a control parameter k. SPH requires the calculation of the viscosity term but since we have implemented a grid-based simulation we left out viscous force. External force is introduced in the next chapter.

4 Method to Make SPH and Level Set Interchangeable

4.1 SPH Bubble Particles in the Subgrid Dynamic

Our hybrid simulation uses the particle level set method. When the value of the ϕ value of marker particles is 0 or less than 0, water is corrected, while more than 0 requires air. However, when marker particles in the particle level set method are left on the opposite side after corrention for level set, they are replaced with escaped particles. Then, in SPH-adjusted simulation method, escaped particles are used to illustrate small scale features of an underlying subgrid dynamically. Buoyancy is determined to make it proportional to each particle volume. Authors of [4,18] used drag force and lift force in the next equation.

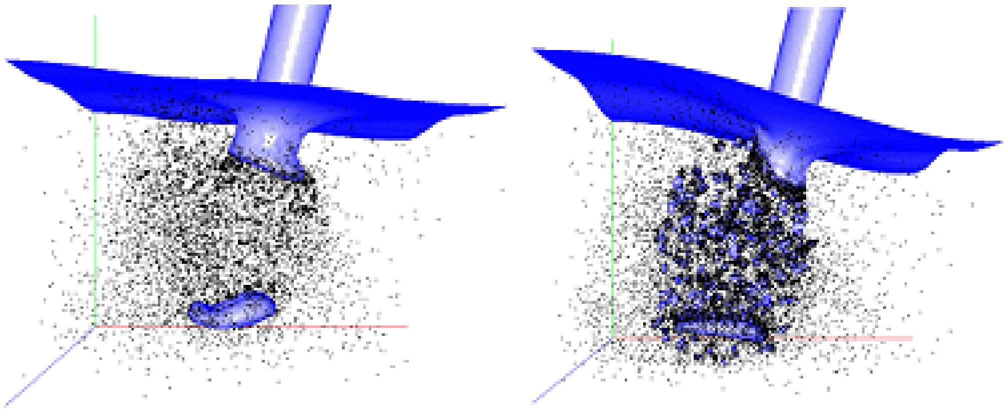

Fig. 2 An example of pouring water. The left denotes SPH particles, showing the motion of the subgrid. Meanwhile, the righ displays air bubbles by converting merged SPH particles into level set. Dots denote SPH particles.

$$f_i^{drag} = -k_{drag} r_i^2 |v_i - u_i| (v_i - u_i) \tag{10}$$

$$f_i^{lift} = -k_{lift} V_i (v_i - u_i) \times \omega_i \tag{11}$$

Where u_i and $\omega_i = \nabla \times u_i$ are the liquid's velocity and vorticity, which are interpolated from the value of the particle's position in the grid; v_i is the particle's velocity. The coefficients k_{drag} and k_{lift} are given in 6000 and 200. Then, the use of lift force was used to make the path of air bubbles unstable. Additionally, Hong et al. [12] proposes SPH vorticity confinement.

$$f_{ij}^{vorticity} = \epsilon \left(N \times \frac{\omega}{|\omega|} \right) \rho_i \tag{12}$$

$\omega = \nabla \times v$ denotes vorticity at the mass center of two SPH particles. Suppose that mass center is p^* and vorticity location vector means $n = p^* - p_i$. Normalizing n becomes $N = n/|n|$. Meanwhile, the greater density ratio of water than that of air induces air bubbles to be merged immediately.

$$f_{ij}^{attraction} = k_{attraction} W_{attraction}(x_{ij}, r_i + r_j) \rho_i \tag{13}$$

Becker and Teschner [2] introduces cohesive attraction force to produce the cluster of air bubbles and shows the physically plausible phenomenon of SPH particles. Attraction force influencing air bubbles to be integrated enhances the ρ of SPH particles which in turn induces rising hi and eventually generates natural bubble turbulence motion in an unstable path when equation (7) pushes adjacent particles and maintains the surface tension of SPH particles.

4.2 Large-scale SPH Particle Back to Grid-based Level Set

We displayed small-scaled bubble motion based on Lagrangian SPH particles in a grid-based simulation to detect detailed features of the subgrid. In fact, the cohesion force SPH particles contain enables particles to be merged, yielding high density and creating air bubbles large enough to be depicted in a grid. Though the creation of SPH particles was originally intended to describe details of a subgrid, the integration of SPH particles larger than subgrid size reduces the simulation accuracy of SPH particles. When the size of SPH particles outgrows that of the subgrid, we turn to grid-based level set.

$$\phi_i^{temp} = \sum mV_j W(x_{ij}, r_j) \tag{14}$$

denotes the level set value of a temporary i node on a grid: m mass coefficient: V_j the volume of particle j. When an SPH particle exists on the i node, it attains temporary value by using the particle on the designated node and recursively checking neighboring nodes and calculating the node value if present. When these nodes are described by grid size, temporary node value is changed into level set value on the grid. In [16] when an SPH particle has a significantly high density, the SPH particle acts as a marker particle of particle level set and back to the level set. However, we render the level set model by SPH particle even when our SPH particle has no marker particle of level set in order for the grid and particle method to faithfully fulfill each intended function: SPH particles in the subgrid method and level set in the grid-based method.

5 Results

These simulations were performed on an Intel Core 2 CPU 3.0GHz and rendered by ray-tracing. The simulation adopted particle level set and BFECC method on a max level 7 octree grid and the radius of particles ranged from 0.3 to 0.8 of a leaf cell. Figure 1 and Figure 2 demonstrate a pouring water simulation. When air bubbles are merged, the left simulation renders particles intact while the right simulation converts larger air bubbles to level set and renders them. Physical dynamics were applied to SPH particles in the subgrid-based simulation and SPH bubble particles above grid size were converted into level set to ensure more accurate simulation. In Figure 3, pouring water on the wall of an empty box creates strong turbulence. In this state, so many escaped particles are formed that the process of transforming SPH particles to level set is clearly shown. The number of SPH particles produced in the simulation shown in Figure 3 stands at 39,435 and a successful simulation is performed to show the natural motion of air bubbles from SPH particles to level set without significant computational cost.

6 Conclusion

In a grid-based fluid simulation using Lagrangian particles, we realistically displayed small-scale details of water and air bubbles which are indefinable on a subgrid. SPH method and physical dynamics facilitated this process. Moreover, we raised the accuracy of simulation by transforming SPH particles larger than subgrid into level set, in

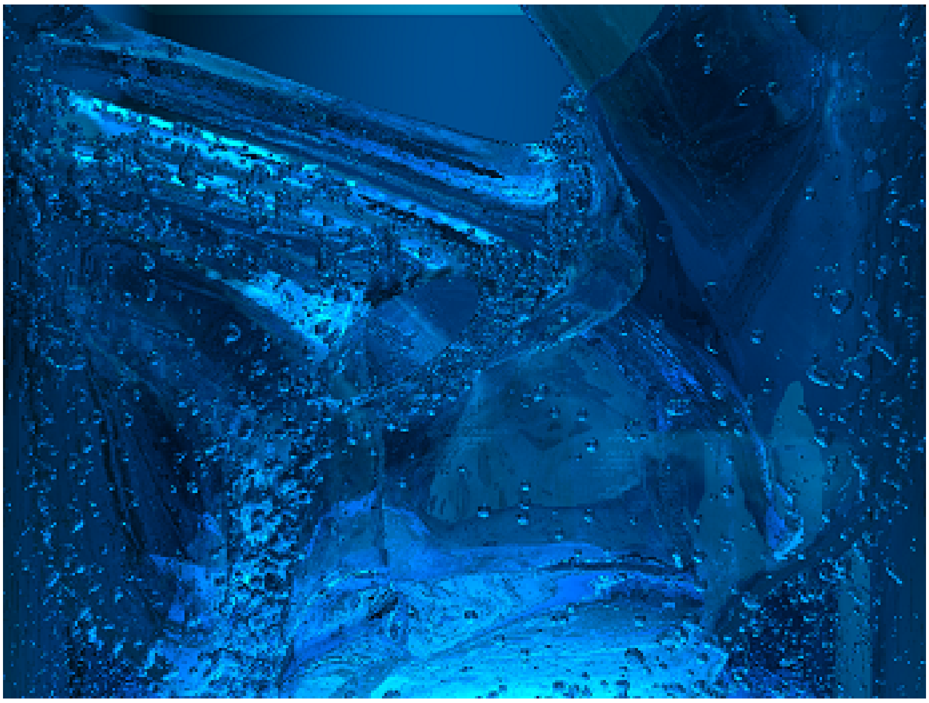

Fig. 3 Pouring water in an empty box generates numerous particles. Air bubbles on the right side of the figure are described in level set from merged particles.

which merged particles become large enough to be depicted on a grid. This hybrid fluid simulation more precisely and naturally illustrates water and air bubbles in multiphase fluid.

References

1. Adams, B., Pauly, M., Keiser. R, Guibas, L. J., Adaptively sampled particle fluids, ACM Trans. Graph. (SIGGRAPH Proc.), 26, 3, 481-487 (2007)
2. Becker, M., Teschner, M., Weakly compressible SPH for free surface flows, In Proc. of 2007 ACM SIGGRAPH/Eurographics Symp. on Comput. Anim., 1-8 (2007)
3. Chorin, A. J., A numerical method for solving incompressible viscous flow problems, Journal of Computational Physics 2, 12-16 (1967)
4. Cleary, P. W., Pyo, S. H., Prakash, M., Koo, B. K., Bubbling and frothing liquids, ACM Trans. Graph. (SIGGRAPH Proc.), 26, 3, 971-976 (2007)
5. Desbrun M., Cani M.-P., Smoothed particles: A new paradigm for animating highly deformable bodies, In 6th Eurographics Workshop on Computer Animation and Simulation, 61-76, (1996)
6. Enright, D., Marschner, S., Fedkiw, R., Animation and rendering of complex water surfaces, ACM Trans. Graph. (SIGGRAPH Proc.)., 21, 3, 736-744 (2002)
7. Foster, N., Fedkiw, R., Practical animation of liquids, In Proc. of ACM SIGGRAPH 2001, 23-30 (2001)
8. Foster, N., Metaxas, D., Realistic animation of liquids, Graph. Models and Image Processing, 58, 471-483 (1996)

9. Greenwood, S. T., House, D. H., Better with bubbles: Enhancing the visual realism of simulated fluid, In Proc. of the 2004 ACM SIGGRAPH/Eurographics Symp. on Comput. Anim., 287-296 (2004)
10. Hong, J.-M., Kim, C.-H., Animation of bubbles in liquid, Comput. Graph. Forum (Eurographics Proc.), 22, 3, 253-262 (2003)
11. Hong, J.-M., Kim, C.-H., Discontinuous fluids, ACM Trans. Graph. (SIGGRAPH Proc.), 24, 3, 915-920 (2005)
12. Hong, J.-M., Lee, H.-Y., Yoon, J.-C., Kim, C.-H., Bubbles Alive, ACM Trans. Graph. (SIGGRAPH Proc.), 48 (2008)
13. Kim, J., Cha, D., Chang, B., Koo, B., Ihm, I., Practical animation of turbulent splashing water, In Proceedings of the 2006 ACM SIGGRAPH/Eurographics Symp. on Comput. Anim., 335-344 (2006)
14. Kim, B., Liu, Y., Llamas, I., Rossignac, J., Flowfixer: Using bfecc for fluid simulation, In Eurographics Workshop on Natural Phenomena 1, 2 (2005)
15. Losasso, F., Gibou, F., Fedkiw, R., Simulating water and smoke with an octree data structure, ACM Trans. Graph. (SIGGRAPH Proc.), 23, 457-462 (2004)
16. Losasso, F., Talton, J., Kwatra, N., Fedkiw, R., Two-Way Coupled SPH and Particle Level Set Fluid Simulation, IEEE Transactions on Visualization and Computer Graphics, 14, 4, 797-804 (2008)
17. Müller, M., Charypar D., Gross M, Particle-based fluid simulation for interactive applications, Proceedings of 2003 ACM SIGGRAPH Symposium on Computer Animation. 154-159 (2003)
18. Magnaudet, J., Eames, I., The motion of highreynolds number bubbles in inhomogeneous flow, Annu. Rev. Fluid Mech. 32, 659-708 (2000)
19. Müller, M., Solenthaler, B., Keiser, R., Gross, M., Particle-based fluid-fluid interaction, In Proc. of the 2005 ACM SIGGRAPH/Eurographics Symp. on Comput. Anim., 237-244 (2005)
20. Stam, J., Stable fluids, In Proc. of ACM SIGGRAPH 1999, 121-128 (1999)
21. Song, O., Shin, H., Ko, H.-S., Stable but nondissipative water, ACM Trans. Graph., 24, 1, 81-97 (2005)
22. Takahashi, T., Fujii, H., Kunimatsu, A., Hiwada, K., Saito, T., Tanaka, K., Ueki, H., Realistic animation of fluid with splash and foam, In EUROGRAPHICS, vol. 22 (2003)