Noname manuscript No.
(will be inserted by the editor)

Simulation of Swirling Bubbly Water using Bubble Particles

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Received: date / Accepted: date

Abstract The effect of surface tension is dynamically and realistically represented within a multiphase fluid simulation. Air bubbles are seeded with 'bubble particles,' which move randomly. These molecule-like movements modify the surface of the air bubbles and generate turbulence in the water. The surface tension between air bubble and water, determined by the composition of the water, remains constant regardless of the size of the bubble, while external forces cause unstable fluid motion as the surface tension strives to remain constant, bubbles split and merge. The bubble particles can also compute for the numerical dissipation usually experienced in grid-based fluid simulations, by restoring the lost volume of individual bubbles. The realistic tearing of bubble surfaces is shown in a range of examples.

Keywords Fluid simulation · Physically based modeling · Bubbles · Grid-based simulation · Multiphase fluids

1 Introduction

Recent fluid animation techniques based on computational fluid dynamics (CFD) have achieved representation of fluid motion which are highly realistic, especially small-scale effects. The appearance of small-scale details such as splashes and bubbles can be enhanced by combining the use of particles with a grid-based simulation. This paper is about the simulation of single bubbles tearing into multiple bubbles within a multiphase fluid. When air is injected from below into a container of water, it forms several bubbles which rise to the surface of the water. Simultaneously, turbulence can be observed in the water. Figure 1 (left) shows a rather simplistic simulation of a bubble

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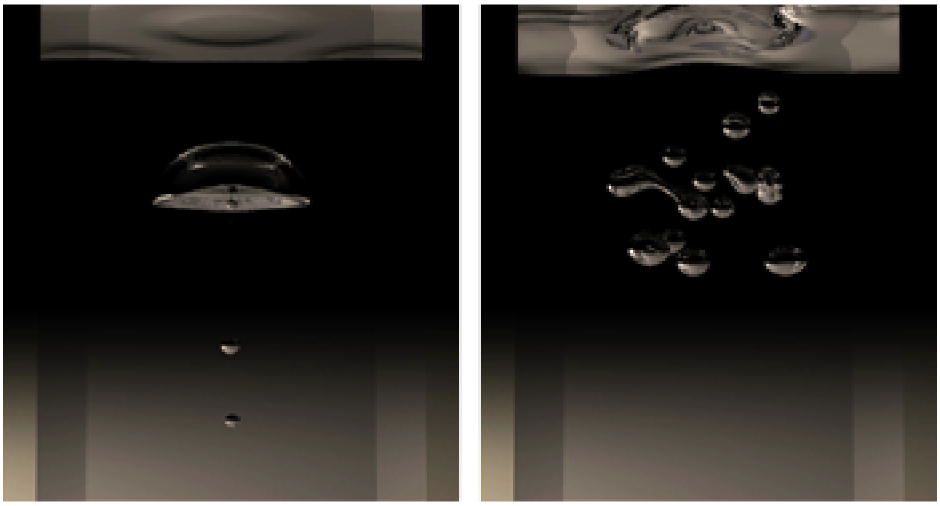

Fig. 1 (left) An air bubble rising in calm water; (right) air bubbles spiraling upwards

rising through calm water. In Figure 1 (right), in contrast, we have simulated turbulence through surface tension, gravity, pressure and other external forces, producing a more complicated and realistic effect. This phenomenon of bubbles breaking up occurs because of a range of different forces that act at the interface between air bubbles and water. To produce a turbulence effect at the bubble interface, we seed bubbles with particles which move randomly, like molecules, and push the level set that represents the wall of a bubble, thereby tearing it into several small bubbles. This method is similar to the vortex particle method in which a dissipated vortex force is added to grid simulations. As well as tearing bubbles, our approach causes the bubbles to spiral under the lift and drag forces produced by the particle dynamics. We also introduce a new fluid simulation in which Eulerian and Lagrangian methods are combined. The movement of the particles allows animators to create a variety of fluid motions by changing the curvature of the bubble walls.

2 Previous Work

Foster and Metaxas [4] were the first to study grid-based 3D fluid simulations using the Navier-Stokes equation; Stam [21] proposed a semi-Lagrangian integration scheme for simulating an unconditionally stable fluid. Other authors [2,3] tracked a smooth water surface using a level set and particles. Takahashi et al. [23] introduced a multiphase simulation in which gas and liquid could be modeled simultaneously; Hong and Kim [7] also simulated multiphase fluids, focusing on the surface tension between bubbles and the liquid. Other multiphase fluid simulations include numerical methods to model surface tension and viscosity chages at interfaces [8], the simulation of a fluids with many components (such as water, air, and fuel) [13], inhomogeneous flow [16], and the small-scale features of multiphase fluids [20]. Shi and Yu [22] controlled a liquid animation using a feedback force; Shin and Kim [19] controlled air bubble in a similar way. Fluid simulations using SPH (smoothed particle hydrodynamics) has been based on

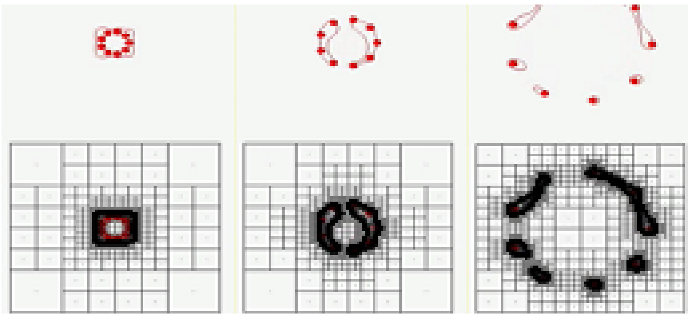

Fig. 2 Level set changes in evolving bubble particles (in two dimensions).

the particle method [17]; the hybrid technique using a grid with particles models small-scale droplets and bubbles [6,10]. Fedkiw et al. [5] introduced vorticity confinement to model the small-scale rolling features characteristic of smoke. Selle et al. presented the vortex particle method [18] to model highly turbulent effects such as explosions or rough water. Hong et al. developed a simulation of bubbly water, in which the sub-grid details were improved by incorporating SPH into a coarse grid [9]. Losasso et al. also improved this technique by coupling a model of dense water volumes to the diffuse regions [14]. These papers represented lively bubble and splash using SPH particles in grid simulation, but we try to simulate turbulent bubbly water using bubble particles.

3 Fluid Simulation

The Navier-Stokes equation provides the simulation of incompressible fluid and preserves mass and momentum.

$$u_t + (u \cdot \nabla)u + \nabla p/\rho = f \quad (1)$$

$$\nabla \cdot u = 0 \quad (2)$$

where ρ is the density of the fluid, and f is external force which includes gravity and control forces. This equation can be solved in two steps. First, intermediate velocity u^*, when pressure is not considered, is solved.

$$u^* = u + \Delta t[-(u \cdot \nabla)u + f] \quad (3)$$

Since $\nabla \cdot u$ should be zero, the pressure profile is determined by solving equation (5); and we get the final velocity profile by equation (4).

$$u = u^* - \frac{\Delta t}{\rho}\nabla p \quad (4)$$

$$\nabla \cdot u = \nabla \cdot u^* - \frac{\Delta t}{\rho}\nabla^2 p = 0 \quad (5)$$

We use an octree [12], to focuses on the interface between the air and water. The particle level set method[2,3] makes this interface a smooth surface.

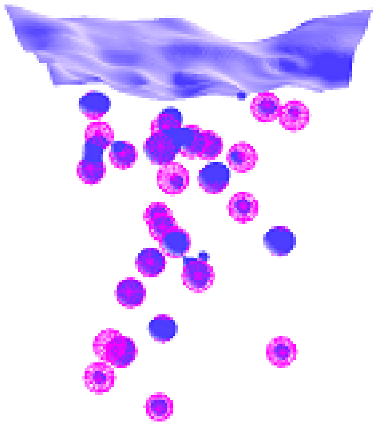

Fig. 3 Bubble particles are seeded in every air bubble; the red spheres in this three dimensional example are bubble particles

4 Bubble Particle Method

Water has tension that minimizes the surface in a free surface; thus air bubbles in water tend to become spherical. However, fluids are also subject to external forces such as gravity, pressure, and turbulence that can tear air bubbles into pieces. We use the seeding of bubble particles to provide a more realistic simulation of bubbles spiraling upward.

4.1 Generation of Bubble Particles

We create bubble particles that move like molecules as follows:

1) Search for air bubbles cell by cell; when a cell with an air bubble is detected, an index is assigned to that cell. If the same air bubble is also found in neighboring cells, the same index is given to those cells. The volume of the air bubble is then calculated. In the case of a cell completely full of air, the volume is the cube of the cell's length. When a cell includes the interface between air and water, the volume is calculated by Monte Carlo integration.

2) Continue this procedure until no air bubble remains without an index.

3) Find a leaf cell in the octree which includes a bubble particle, and assign the index of that cell to that particle.

4) If there is no bubble particle in an air bubble, new bubble particles are seeded within that bubble.

5) If bubble particles with different indices are found within one air bubble, it means two air bubbles have merged. So the bubble particles should be given the same index and the volume of the air bubble should be recalculated. When bubble particles with the same index are found in different air bubbles, it means the air bubble has split. In this case the bubble particles should be given another index and the volume should be recalculated as well.

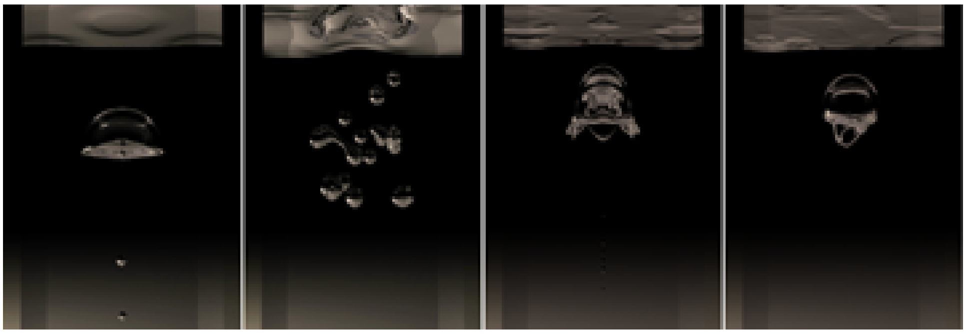

Fig. 4 Images from left to right : base simulation, bubble particle method, vorticity confinement method, and vortex particle method

4.2 Movement of a Bubble Particle

A bubble particle moves in an air bubble with a random path like a molecule exhibiting Brownian motion. Bubble particle is given an initial velocity and its subsequent movement is irregular. We model this irregular motion as follows :

$$u_{bp}^{n+1} = u_{bp}^{n} + k\nabla\phi,\ \ \phi \leq r_{bp} \tag{6}$$

where u_{bp}^{n} is a bubble particle's velocity in the current time step; u_{bp}^{n+1} is the bubble particle's velocity in the next time step; k is the coefficient of the degree to which the bubble particle is pushed toward a higher gradient of ϕ, so the bubble particle bounces against the interface between the air and water, evolving like a molecule. ϕ is the level set value of a bubble particle's position. r_{bp} is a bubble particle's radius. If a bubble particle is near the interface between water and bubble, that is, the level set value of the bubble particle's position is almost zero, the bubble particle turns in the direction of the level set's gradient of its position, causing it to move as a molecule does. Authors of [15, 1] simulated using drag force ($f^{drag} = \frac{1}{2}\rho\pi r^2|v^{particle} - u^{liquid}|(v^{particle} - u^{liquid})$) and lift force ($f^{lift} = -\rho V(v^{particle} - u^{liquid}) \times \nabla \times u$). The drag force offset the particle's velocity by the velocity of liquid; the lift force generates vortex force around the particle's position. These forces cause the spiraling movement of an air bubble, but in their simulations, the air bubble was made of a particle. This can be seen in the interaction between the air bubble and water. In contrast, in this paper, air bubbles move upward, while swirling and changing level set, which makes the simulation appear more realistic.

4.3 The Effect of a Bubble Particle on the Surface of an Air Bubble

A bubble particle affects the velocity of its neighbors in the grid, so the level set of the fluid and the surface of air bubbles change. Consequently, air bubbles split while spiraling. The neighboring velocity's affection equation by bubble particle's velocity is :

$$u_{nb}^{n+1} = u_{nb}^{n} + su_{bp}^{n},\ \ |P_{nb} - P_{bp}| \leq r_{bp} \tag{7}$$

where u_{nb}^{n+1} is a neighboring node's new velocity, u_{nb}^{n} is the neighboring node's current velocity, s is a coefficient of a neighboring node's velocity pushed by a bubble particle, u_{bp}^{n} is a bubble particle's velocity. P_{nb} is a neighboring node's position, and P_{bp} is a bubble particle's position. This process should be performed before the projection step. The bubble surface is changed by updating neighboring node's velocities; the bubble tears into several smaller bubbles and rise in a zigzag fashion. The vortex particle method [18] also attempted to make turbulence, but it did not present bubbles tearing at their rims. We assist bubble particles to generate Rayleigh instability phenomenon; air bubbles appear alive without the lift and drag forces of a particle system.

5 Correcting air Bubble volumes

In grid-based fluid simulations, advection causes the dissipation of air bubble volumes; the larger the size of a cell in the grid and the time step, the bigger this dissipation becomes. When an air bubble moves up that is smaller than the size of a cell, it disappears, in most cases, by numerical dissipation before reaching the surface. BFECC and volume control method [11] have been studied in order to significantly reduce this numerical dissipation. We also correct bubbles' volume with a similar method. We got bubble volume in chapter 4.1, and assumed V_i^n is the volume of index i at n_{th} time step. If there is no dissipation of an air bubble, $V_i^n = V_i^{n-1}$; if there is dissipation, when x_i^n is the volume dissipated, $x_i^n = V_i^n$ - V_i^{n-1}. The volume dissipated should be compensated for in the air bubbles; divergence value $c_i^n = -k_p x_i^n$. Where k_p is the bubble loss compensation coefficient. This divergence value c_i^n is calculated in the projection step.

In a cell without bubble particles

$$\nabla^2 p = \frac{\rho}{\Delta t}(\nabla \cdot u^*) \tag{8}$$

and in a cell that contains bubble particles

$$\nabla^2 p = \frac{\rho}{\Delta t}(\nabla \cdot u^* - c_i^n) \tag{9}$$

When the projection step is calculated as above, the air bubble with bubble particles is compensated for its loss of volume. Although the compensation solution in this paper is similar with that of [11], they differ in the following ways: first whereas [11] compensated for the loss of whole fluid, our simulation focuses on air bubbles, only compensating for the volume loss of air bubbles; second, our method also checks the mergers and the splits of air bubbles by bubble particles. This is its strength.

6 Results

Simulations were performed on a PC with an Intel Core2 CPU 3.0, and rendered by ray-tracing. In Figure 4, we compare bubble particle method with base simulation, vorticity confinement method, and vortex particle method. The bubble particle method shows swirling bubbly water while other simulations fail to present tearing at the rim of air bubbles. The extra computational cost incurred by using bubble particle was about 10% overhead. Figure 5 shows air bubbles interacting with a rigid sphere; In Figure 6,

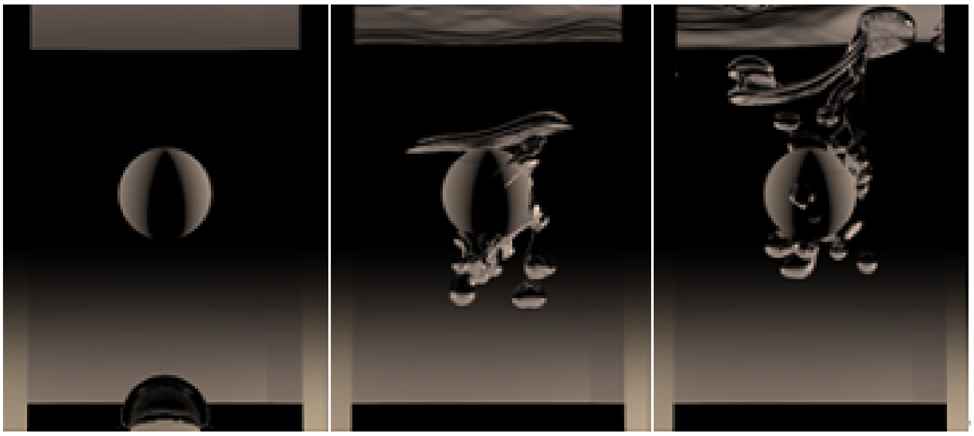

Fig. 5 Rising air bubbles with bubble particles interacting with a rigid sphere

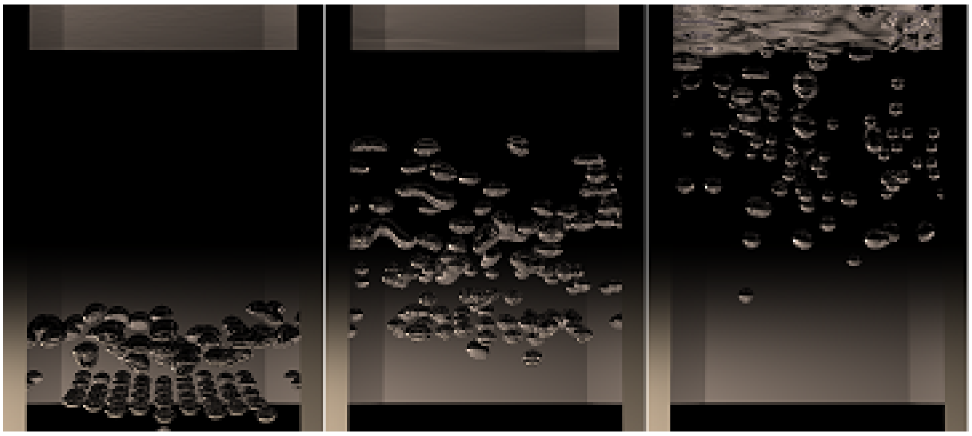

Fig. 6 Bubbles zigzagging as they rise

Table 1 Comparison of simulation time bubble particle method with base simulation, vorticity confinement, and vortex particle method in 128×256×128 grid simulation

base simulation	bubble particle method	vorticity confinement	vortex particle
82.1sec/frame	91.7 sec/frame	85.2 sec/frame	88.4 sec/frame

bubble particles, seeded in air bubbles, change the level set value by moving, thereby creating lively fluid motions in calm water.

7 Conclusions

We seeded Lagrangian bubble particles in air bubbles and caused them to move like molecules in order to create turbulence and consequently simulate realistic fluid in a grid based fluid simulation. Large air bubbles tear into several small bubbles; small bubbles zigzag. When the Rayleigh instability phenomenon appears in this simulation, the flow field must be considered in future work. We removed the dissipation of air bubbles by making bubble particles check the volumes of air bubbles. In conclusion, the contributions of this paper is that it enables animators to control various fluid motions by creating turbulence with bubble particles and correcting the volume of air bubbles

Acknowledgements This work was supported by the Second Brain Korea 21 Project. This work was supported by the IT Research and Development program of MKE/IITA. [2006-S-033-03, Development of wearable interface technology for emergency rescue]. J.H. was supported by the Korea Science and Engineering Foundation(KOSEF) grant funded by the Korea government(MOST)(No. R01-2008-000-11301-0).

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